

KUNS 1388
 HE(TH) 96/02
 hep-th/9605017

On thermodynamics of black p -branes

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May, 1996

Abstract

Thermodynamic properties of a class of black p -branes in D -dimensions considered by Duff and Lu are investigated semi-classically. For black $(d - 1)$ -brane, thermodynamic quantities depend on D and d only through the combination $\tilde{d} \equiv D - d - 2$. The behavior of the Hawking temperature and the lifetime vary with \tilde{d} , with a critical value $\tilde{d} = 2$. For $\tilde{d} > 2$, there remains a remnant, in which non-zero entropy is stored. Implications of the fact that the Bekenstein-Hawking entropy of the black $(d - 1)$ -brane depend only on $\tilde{d} = D - d - 2$ is discussed from the point of view of duality.

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1 Introduction

It has been known that there is a striking analogy between the laws of black hole and the laws of thermodynamics. However it has raised problems such as statistical description of the entropy and the information loss paradox.

Recently, Strominger and Vafa calculated the entropy of a certain extremal black hole microscopically using D-brane technology [1]. After that, some attempts have been made to extend their calculation to other solutions and non-extremal limit [2]. By inspecting these investigations, it seems that the constancy of a dilaton is a key ingredient for the D-brane interpretation of the entropy to be applicable, as pointed out in [3].

However, various kinds of black p -branes with a non-constant dilaton are known. So, it is interesting to study whether some microscopic explanation of the entropy is possible for such black p -branes. As a step toward this direction, we investigate thermodynamic properties of a class of dilatonic black p -branes which are considered by Duff and Lu [4, 5], semi-classically.

The action of this model includes an antisymmetric tensor potential of rank d in D -dimensional spacetime, interacting with gravity and a dilaton, and it represents a part of the low energy effective actions of typeIIA, typeIIB superstrings and M-theory. There are two types of black p -brane solutions which are related by duality: black $(d-1)$ -brane with an electric charge and black $(\tilde{d}-1)$ -brane with a magnetic charge. It turns out that in the expression of thermodynamic quantities of the $(d-1)$ -brane, D and d appear only through the combination $\tilde{d} = D - d - 2$, hence the thermodynamic properties vary with \tilde{d} . As the black $(d-1)$ -brane approaches to the extremal limit, the Hawking temperature goes to zero for $\tilde{d} > 2$, while it goes to infinity for $\tilde{d} < 2$. For $\tilde{d} = 2$, the temperature is finite at this limit. Thus thermodynamic property of the black $(d-1)$ -brane changes by the sign of $\tilde{d} - 2$.

This feature also appears in lifetime of the black $(d-1)$ -brane. By assuming that the black $(d-1)$ -brane radiates just like a black body of the Hawking temperature, and loses its mass due to the radiation of energy, we calculate the lifetime, that is, the time until the mass of the black $(d-1)$ -brane decreases to the extremal limit. It is finite for $\tilde{d} > 2$ and infinite for $\tilde{d} < 2$. It can be interpreted that non-zero entropy is stored in the remnant $(d-1)$ -brane for $\tilde{d} < 2$.

In section 2, we review the black p -brane solutions discussed by Duff and Lu. Thermodynamic properties of the black p -brane are investigated in section 3. In section 4, implications of the fact that the black $(d-1)$ -brane depend only on \tilde{d} are discussed by taking duality into account.

2 Black p-branes in string theory

In this section, we recapitulate the black p -brane solutions discussed in [4, 5]. We consider the following D -dimensional action

$$S = \frac{1}{16\pi} \int d^D x \sqrt{-g} \left[R - \frac{1}{2}(\partial\phi)^2 - \frac{1}{2(d+1)!} e^{-\alpha\phi} F_{d+1}^2 \right] \quad (2.1)$$

where ϕ denotes a dilaton, and F_{d+1} is a field strength for an antisymmetric tensor A_d of rank d , i.e.

$$F_{d+1} = dA_d. \quad (2.2)$$

We take the constant α to satisfy

$$\alpha^2 = 4 - \frac{2d(D-d-2)}{D-2}. \quad (2.3)$$

This choice of α has a following significance. Bosonic part of low energy effective actions of typeIIA and typeIIB superstrings are

$$\begin{aligned} I_{\text{IIA}} &= \frac{1}{16\pi} \int d^{10} x \sqrt{-g} \left[R - \frac{1}{2}(\partial\phi)^2 - \frac{1}{4} e^{\frac{3}{2}\phi} F_2^2 - \frac{1}{12} e^{-\phi} F_3^2 - \frac{1}{48} e^{\frac{1}{2}\phi} F_4^2 \right] + \dots \\ I_{\text{IIB}} &= \frac{1}{16\pi} \int d^{10} x \sqrt{-g} \left[R - \frac{1}{2}(\partial\phi)^2 - \frac{1}{12} e^{-\phi} F_3^{(1)2} - \frac{1}{12} e^{\phi} F_3^{(2)2} \right] + \dots \end{aligned} \quad (2.4)$$

where self-dual five-form of typeIIB theory is set to zero. Here various kinds of field strength appear, but the coefficient of the dilaton satisfies the equation (2.3) for each value of d . In addition, bosonic part of eleven-dimensional supergravity, which is considered as the low energy effective action of M-theory,

$$I_{\text{M}} = \frac{1}{16\pi} \int d^{10} x \sqrt{-g} \left[R - \frac{1}{12} F_3^2 \right] + \dots \quad (2.5)$$

also contains the action (2.1): in this case, $\alpha = 0$, and hence ϕ decouples.

The equations of motion for the action (2.1) are

$$\nabla^M (e^{-\alpha\phi} F_{MM_1\dots M_d}) = 0, \quad (2.6)$$

$$\square\phi + \frac{\alpha}{2(D-2)!} e^{-\alpha\phi} F_{d+1}^2 = 0, \quad (2.7)$$

$$\begin{aligned} R_{MN} &= \frac{1}{2} \nabla_M \phi \nabla_N \phi + \frac{2}{(D-3)!} e^{-\alpha\phi} F_{MM_1\dots M_d} F_N^{M_1\dots M_d} \\ &\quad - \frac{1}{2(D-2)(D-2)!} e^{-\alpha\phi} g_{MN} F_{d+1}^2 \end{aligned} \quad (2.8)$$

where $M, N = 0, 1, \dots, D - 1$.

The black $(d - 1)$ -brane solution for these equations is

$$ds^2 = - \left[1 - \left(\frac{r_+}{r} \right)^{\tilde{d}} \right] \left[1 - \left(\frac{r_-}{r} \right)^{\tilde{d}} \right]^{-\frac{\tilde{d}}{d+\tilde{d}}} dt^2 + \left[1 - \left(\frac{r_+}{r} \right)^{\tilde{d}} \right]^{-1} \left[1 - \left(\frac{r_-}{r} \right)^{\tilde{d}} \right]^{-1+\frac{\alpha^2}{2\tilde{d}}} dr^2 \\ + r^2 \left[1 - \left(\frac{r_-}{r} \right)^{\tilde{d}} \right]^{\frac{\alpha^2}{2\tilde{d}}} d\Omega_{\tilde{d}+1}^2 + \left[1 - \left(\frac{r_-}{r} \right)^{\tilde{d}} \right]^{\frac{\tilde{d}}{d+\tilde{d}}} \delta_{ij} dx^i dx^j, \quad (2.9)$$

$$A_{01\dots d-1} = \left(\frac{r_+ r_-}{r^2} \right)^{\frac{\tilde{d}}{2}}, \quad (2.10)$$

$$e^{-2\phi} = \left[1 - \left(\frac{r_-}{r} \right)^{\tilde{d}} \right]^{-\alpha} \quad (2.11)$$

where $i, j = 1, 2, \dots, d - 1$, $d\Omega_n^2$ is the metric on unit n -sphere and we define

$$\tilde{d} = D - d - 2. \quad (2.12)$$

For this solution, field strength is expressed as

$$e^{-\alpha\phi} * F_{d+1} = \tilde{d}(r_+ r_-)^{\frac{\tilde{d}}{2}} \epsilon_{\tilde{d}+1} \quad (2.13)$$

where $\epsilon_{\tilde{d}+1}$ is a volume form on unit $(\tilde{d} + 1)$ -sphere.

This solution has an event horizon at $r = r_+$, and an inner horizon at $r = r_-$ for $r_+ > r_-$. $r_+ = r_-$ corresponds to the extremal BPS state.

This black $(d - 1)$ -brane has an "electric" charge with respect to the rank $(d + 1)$ field strength,

$$Q = \frac{1}{16\pi} \int_{S^{\tilde{d}+1}} e^{-\alpha\phi} * F_{d+1} = \frac{1}{16\pi} \Omega_{\tilde{d}+1} \tilde{d}(r_+ r_-)^{\frac{\tilde{d}}{2}} \quad (2.14)$$

where Ω_n is the volume of unit n -sphere.

In order to obtain black p -branes with a "magnetic" charge, we use the fact that the equations of motion (2.6)-(2.8) are invariant under the following duality transformation

$$d \rightarrow \tilde{d}, \quad (2.15)$$

$$\alpha \rightarrow -\alpha, \quad (2.16)$$

$$e^{-\alpha\phi} * F_{d+1} \rightarrow F_{d+1}. \quad (2.17)$$

By applying this transformation to the "electrically" charged black $(d - 1)$ -brane, we get a "magnetically" charged black $(\tilde{d} - 1)$ -brane,

$$ds^2 = - \left[1 - \left(\frac{r_+}{r} \right)^d \right] \left[1 - \left(\frac{r_-}{r} \right)^d \right]^{-\frac{\tilde{d}}{d+\tilde{d}}} dt^2 + \left[1 - \left(\frac{r_+}{r} \right)^d \right]^{-1} \left[1 - \left(\frac{r_-}{r} \right)^d \right]^{-1+\frac{\alpha^2}{2\tilde{d}}} dr^2$$

$$+r^2 \left[1 - \left(\frac{r_-}{r} \right)^d \right]^{\frac{\alpha^2}{2d}} d\Omega_{d+1}^2 + \left[1 - \left(\frac{r_-}{r} \right)^d \right]^{\frac{d}{d+1}} \delta_{ij} dx^i dx^j, \quad (2.18)$$

$$F_{d+1} = d(r_+ r_-)^{\frac{d}{2}} \epsilon_{d+1}, \quad (2.19)$$

$$e^{-2\phi} = \left[1 - \left(\frac{r_-}{r} \right)^d \right]^\alpha \quad (2.20)$$

where $i, j = 1, 2, \dots, \tilde{d} - 1$.

3 Thermodynamics of black p-branes

In this section, we investigate thermodynamic properties of the black $(d-1)$ -brane. In the following, we consider quantities per unit $(d-1)$ -volume for extensive ones. If we rewrite the metric of the "electric" $(d-1)$ -brane (2.9) as

$$ds^2 = -A(r)dt^2 + B(r)dr^2 + r^2 C(r) d\Omega_{\tilde{d}+1}^2 + D(r) \delta_{ij} dx^i dx^j, \quad (3.1)$$

ADM mass of the black $(d-1)$ -brane is calculated following [6],

$$\begin{aligned} M &= -\frac{1}{16\pi} \Omega_{\tilde{d}+1} \left[(\tilde{d}+1) r^{\tilde{d}+1} \partial_r C + (d-1) r^{\tilde{d}+1} \partial_r D - (\tilde{d}+1) r^{\tilde{d}} (B-C) \right]_{r \rightarrow \infty} \\ &= \frac{1}{16\pi} \Omega_{\tilde{d}+1} \left[(\tilde{d}+1) r_+^{\tilde{d}} - r_-^{\tilde{d}} \right]. \end{aligned} \quad (3.2)$$

At the extremal limit $r_+ = r_-$, the mass coincides with the charge Q (2.14).

The Bekenstein-Hawking entropy of the black $(d-1)$ -brane is one forth of the area A_{EH} of the event horizon,

$$S = \frac{1}{4} A_{\text{EH}} = \frac{1}{4} \left[(r^2 C)^{\frac{\tilde{d}+1}{2}} D^{\frac{d-1}{2}} \right]_{r=r_+} = \frac{1}{4} \Omega_{\tilde{d}+1} r_+^{\tilde{d}+1} \left[1 - \left(\frac{r_-}{r_+} \right)^{\tilde{d}} \right]^{\frac{\tilde{d}+2}{2d}}. \quad (3.3)$$

(We have set the Newton constant one.) We note that the entropy vanishes at the extremal limit.

The Hawking temperature can be computed by analytically continuing in t and requiring that the resulting Riemannian space has no conical singularity. This requires a periodic identification in imaginary time and the temperature is inverse of the period. For the black $(d-1)$ -brane,

$$T = \frac{1}{4\pi} \left[\sqrt{\partial_r A \partial_r \left(\frac{1}{B} \right)} \right]_{r=r_+} = \frac{\tilde{d}}{4\pi r_+} \left[1 - \left(\frac{r_-}{r_+} \right)^{\tilde{d}} \right]^{\frac{\tilde{d}-2}{2d}}. \quad (3.4)$$

Chemical potential associated with the electric charge (2.14) is the value of the gauge potential at the horizon,

$$\mu = A_{01\dots d-1}|_{r=r_+} = \left(\frac{r_-}{r_+}\right)^{\frac{\tilde{d}}{2}}. \quad (3.5)$$

We can verify that these quantities satisfy the first law of thermodynamics

$$dM = TdS + \mu dQ. \quad (3.6)$$

In order to obtain thermodynamic quantities for the "magnetic" $(\tilde{d} - 1)$ -brane, we only need to exchange d and \tilde{d} .

Now we discuss thermodynamic properties of the "electric" $(d - 1)$ -brane. What is the most remarkable thing is that in the expression of thermodynamic quantities, " d " does not appear explicitly when we write $D = d + \tilde{d} - 2$. We discuss this point in section 4. Next, we can see that the behavior of the temperature varies with the value \tilde{d} with a critical value $\tilde{d} = 2$. At the extremal limit, $M = Q$, the temperature is zero and finite for $\tilde{d} > 2$ and $\tilde{d} = 2$, respectively. For $\tilde{d} < 2$, the temperature goes to infinity as the mass approaches to the extremal limit.

The fact that $\tilde{d} = 2$ is critical also appears in lifetime of the black $(d - 1)$ -brane. We assume that the black $(d - 1)$ -brane radiates just like a black body of temperature T , and the back reaction effect is to cause the black $(d - 1)$ -brane to lose mass at the same rate at which energy is radiated to infinity. To estimate the order of the magnitude of the evaporation rate, we consider the thermal radiation of massless fields. The energy density of massless fields in thermal equilibrium at temperature T is proportional to T^D in D -dimensional spacetime (Stefan Boltzmann's law). So the energy flux from the black $(d - 1)$ -brane is proportional to $T^D A_{\text{EH}}$. If we let the deviation of the mass from the extremal limit as δM , the Hawking temperature T and the area A_{EH} of the event horizon behaves as

$$T \sim (\delta M)^{\frac{\tilde{d}-2}{2\tilde{d}}}, \quad (3.7)$$

$$A_{\text{EH}} \sim (\delta M)^{\frac{\tilde{d}+2}{2\tilde{d}}} \quad (3.8)$$

to leading order in δM . Therefore δM satisfies the following differential equation

$$\frac{d(\delta M)}{dt} \sim -(\delta M)^{\frac{\tilde{d}-2}{2\tilde{d}}D + \frac{\tilde{d}+2}{2\tilde{d}}}. \quad (3.9)$$

If $\tilde{d} = 2$, the solution takes the form

$$\delta M \sim e^{-at} \quad (3.10)$$

with some positive constant a , that is, the mass decreases exponentially to the extremal limit. If $\tilde{d} \neq 2$, the solution takes the form,

$$\delta M \sim \left[\frac{(D-1)(\tilde{d}-2)}{2\tilde{d}}t + b \right]^{-\frac{2\tilde{d}}{(D-1)(\tilde{d}-2)}} \quad (3.11)$$

with some positive constant b . As $D - 1$ is positive, the behavior of δM changes by the sign of $\tilde{d} - 2$. For $\tilde{d} < 2$, δM becomes zero in finite time like Schwarzschild black hole in four dimensional spacetime. For $\tilde{d} > 2$, it takes infinite time until the mass of the black $(d - 1)$ -brane decreases to the extremal limit. It can be interpreted that there remains a remnant, in which non-zero entropy is stored.

4 Discussion

In this section, we discuss implications of the fact that thermodynamic quantities, especially, the Bekenstein-Hawking entropy, of the black $(d - 1)$ -brane depend on d and D only through the combination $\tilde{d} = D - d - 2$. In the process of the calculation, they are rather complicated expressions in d and \tilde{d} . After the calculation, however, they depend only on \tilde{d} . We would like to stress that this is not so trivial. If the spacetime is a direct product of a $\tilde{d} + 3$ -dimensional black hole and R^{d-1} , it is a natural result that quantities per unit $(d - 1)$ -volume are independent of d . But this is not the case. Above all, this is a characteristic feature of the choice of α (2.3). It is reasonable to think that this fact is not accidental but has some physical explanation.

Of course, quantum effects correct the solution in general, since the black $(d - 1)$ -brane does not corresponds to BPS states. So we have no right to trust this solution away from the extremal limit, but it is difficult to resist speculating about what it might mean if the solution had a similar property after including quantum corrections.

Now we take the duality into account. As explained in section 2, there is a duality which interchanges d and \tilde{d} , but d and \tilde{d} are not equivalent in the sense that \tilde{d} is a function of d and D , that is,

$$\tilde{d} = \tilde{d}(d, D). \quad (4.1)$$

However, from the spirit of duality, it is natural to consider that d and \tilde{d} are totally equivalent, hence the dimension of spacetime D is a function of d and \tilde{d} ,

$$D = D(d, \tilde{d}), \quad (4.2)$$

where d and \tilde{d} are independent.

In other word, it is natural to consider that $(d - 1)$ -brane and $(\tilde{d} - 1)$ -brane exist at first, and the (uncompactified) dimension of spacetime is determined as the dimension in which $(d - 1)$ -brane and $(\tilde{d} - 1)$ -brane are dual, that is, they are related by the Dirac quantization condition.

This way of thinking leads to a rather strange conclusion that the origin of the entropy of the black $(d - 1)$ -brane, which depends only on \tilde{d} , consists of its dual partner, $(\tilde{d} - 1)$ -brane.

As we noted, however, our discussion is based on semi-classical treatment of the non-extremal black p -branes. In order to clarify that such an interpretation have some sugges-

tion to the understanding of black p -branes, it is necessary to take quantum corrections into account ¹.

Finally, we note that we can not have an extremal black p -brane with non-zero entropy even if we boost the solution to one of the directions of the p -brane as in [2].

Acknowledgements

I would like to thank S. Yahikozawa for careful reading of this manuscript. This work is in part supported by Soryushi Shogakukai.

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¹While this paper was prepared, we received preprint [7] which attempts to take one loop corrections into account for dilatonic black p -branes.